Nonlinear Analysis of Optimized Structure with Constraints on System Stability

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An optimization algorithm based on an optimality criterion was used to design a minimum weight space truss with different constraint requirements on system stability. The constraints were specified so that the eigenvalues associated with all the critical buckling modes are either equal or separated by a specified factor. For the second case the critical buckling mode was preselected from all the possible critical modes. The designs obtained for the various constraint conditions were analyzed with and without specified geometric imperfections using a nonlinear finite element program which accounts for geometric nonlinearity. The results obtained for various designs were compared for their imperfection sensitivity.

Introduction

IN an optimum structural design problem the design variables are selected so as to minimize the weight of the structure and satisfy all the constraints. The constraints may include maximum allowable stresses in the elements, displacement limits at the node points, frequency requirements, system stability, gage limitations, etc. Depending on the nature of the loads applied to the structure and the geometry, one or more than one of these constraints can be active and control the design of the structure. The cross-sectional areas of the members will generally be dictated by the dominant constraints. In space structures, system stability can become a critical constraint and the structure will have to be designed to satisfy this constraint. The system stability requirement may be defined by an eigenvalue problem. In designing a structure with this requirement more than one critical buckling mode can be active at the optimum. A design where the buckling load associated with more than one critical buckling mode is the same is known as a simultaneous mode design. These designs tend to become imperfection sensitive. A small change or deviation in the geometry of the structure can substantially reduce the load carrying capacity of the structure. It may be possible to reduce the imperfection sensitivity of a structure by designing the structure so that the eigenvalues associated with the system buckling modes are separated by a specified interval. The purpose of this paper is to investigate this phenomena by optimizing a structure with various system stability constraint requirements and then analyzing them with specific imperfections.

The degrading effect of geometric imperfections on optimized structures with the design variables as the geometry of the structure or structures optimized to fail in more than one buckling mode has been extensively discussed in Refs. 1-4. It has been pointed out in Ref. 4 that if the structure is optimized so that the lowest buckling loads due to local and overall buckling do not occur at the same load level, then the structure becomes less imperfection sensitive. The imperfection sensitivity can be reduced by allowing the local buckling to occur before the overall buckling of the structure. The structures investigated in these references are primarily shell or plate type, and the modes of failure were system and local buckling.

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The problem considered in this system is a three-dimensional dome structure idealized with bar elements. The structure has been optimized by using an algorithm based on an optimality criterion with constraints on system stability. The nonlinear response of the optimized structure with and without geometric imperfections is determined by solving the nonlinear equilibrium equations by the Newton-Raphson method at different load levels.

Optimization Method

The algorithm based on an optimality criterion[5] uses a recurrence relation to modify the design variables. The recurrence relation is derived from the optimality criterion. A short derivation of the optimality criterion and the algorithm is given here. A detailed discussion may be found in Ref. 6.

The optimization problem can be defined as follows: Minimize the weight of the structure,

$$W=\sum_{i=1}^{n} \rho_i A_i \ell_i \tag{1}$$

subjected to

$$g_j=\mu_j-\alpha_j\bar{\mu}\geq 0 \tag{2}$$

where ρ_i is the density of the material, A_i the design variable, ℓ_i the volume parameter for unit value of the design variable A_i, n the number of elements, g_j the jth constraint, μ_j the eigenvalue associated with the jth constraint, α_j the factor separating the eigenvalues, and $\bar{\mu}$ the lowest eigenvalue which will be equal to μ_1 if the eigenvalues are arranged in the ascending order.

The eigenvalue μ_j can be determined by solving the equation

$$\{[K]-\mu_j[K_G]\}\{\psi_j\}=0 \tag{3}$$

where $[K]$ is the linear total stiffness matrix of the structure, $[K_G]$ the geometric stiffness matrix of the structure which is a function of the internal force distribution due to the applied load $\{P\}$, and $\{\psi_j\}$ is the eigenvector associated with the eigenvalue μ_j. The critical eigenvalue is the lowest eigenvalue which is equal to μ_1. The buckling load of a structure is given by $\mu_1\{P\}$.

Using Eqs. (1) and (2), the Lagrangian can be written as

$$L(A,\lambda)=\sum_{i=1}^{n}\rho_i A_i \ell_i-\sum_{j=1}^{m}\lambda_j(\mu_j-\alpha_j\bar{\mu}) \tag{4}$$

where λ_j are the Lagrange multipliers. Differentiating this equation with respect to the design variables A_i and setting the resulting equations to zero gives

$$\rho_i \ell_i - \sum_{j=1}^{m} \lambda_j \frac{\partial \mu_j}{\partial A_i} = 0 \tag{5}$$

The gradient of the eigenvalue is given by

$$\frac{\partial \mu_j}{\partial A_i} = \frac{1}{A_i} \frac{\{\psi_j\}_i^t [k]_i \{\psi_j\}_i}{\{\psi_j\}[K_G]\{\psi_j\}} \tag{6}$$

where $[k]_i$ is the stiffness matrix of the ith element and $\{\psi_j\}_i$ is the component of the jth buckling mode associated with the ith element. Equation (6) can be written as

$$\frac{\partial \mu_j}{\partial A_i} = \frac{B_{ij}}{A_i^2} \tag{7}$$

where

$$B_{ij} = A_i \{\tilde{\psi}_j\}_i^t [k]_i \{\tilde{\psi}_j\}_i \tag{8}$$

In this equation $\{\tilde{\psi}_j\}_i$ is the normalized buckling mode associated with ith element given by

$$\{\tilde{\psi}_j\}_i = [\{\psi_j\}^t [K_G]\{\psi_j\}]^{-1/2}\{\psi_j\} \tag{9}$$

Substituting Eq. (9) in Eq. (5) the optimality criterion can be written as

$$1 = \sum_{j=1}^{m} \lambda_j \frac{B_{ij}}{A_i^2 \rho_i \ell_i} \qquad i = 1,\ldots,n \tag{10}$$

The Lagrange multipliers λ_j in this equation must satisfy the condition

$$\lambda_j(\mu_j - \alpha_j \bar{\mu}) = 0 \qquad j = 1,\ldots,m \tag{11}$$

Different recurrence relations can be derived from Eq. (10). In this paper the recurrence relation

$$A_i^{\nu+1} = A_i^{\nu}\left[1 - \frac{1}{r}\left(\sum_{j=1}^{m} \lambda_j \frac{B_{ij}}{A_i^2 \rho_i \ell_i} - 1\right)\right]^{-1} \tag{12}$$

is used. In this equation $\nu+1$ and ν are the iteration numbers and r is the parameter that controls the step size. In order to use this relation it is required to evaluate the coefficients B_{ij} and the Lagrange multipliers. The coefficients B_{ij} can be evaluated by using Eq. (8). The Lagrange multipliers can be determined by solving the set of simultaneous equations

$$\sum_{k=1}^{m} \lambda_k^{\nu+1} \left(\sum_{i=1}^{n} \frac{B_{ij} B_{ik}}{\rho_i \ell_i A_i^3} \right)_{\nu} = r(\alpha_j \bar{\mu} - \mu_j^{\nu}) + \mu_j^{\nu} \quad j = 1,\ldots,m \tag{13}$$

These equations are derived by enforcing the condition that the change in the design variables should satisfy the constraint relations.

In design problems the load applied to the structure is known. The lowest buckling load will be equal to the applied load if the lowest eigenvalue is equal to unity. This requires scaling of the design. The scaling parameter Λ is given by

$$\Lambda = 1/\mu_1' \tag{14}$$

where μ_1' is the lowest eigenvalue determined for unscaled design.

Nonlinear Analysis

The type of nonlinearity we are going to consider is geometric nonlinearity. This occurs when the deflections of the node points are large enough to cause significant changes in the geometry of the structure. This requires formulation of the equilibrium equations with respect to the deformed geometry. The incremental form of the nonlinear equilibrium equations in matrix form can be written as

$$\{[K_T] + [K_G]\}\{X\} = \{F\} - \{I\} \tag{15}$$

where $[K_T]$ is the tangent stiffness matrix, $[K_G]$ the geometric stiffness matrix, $\{X\}$ the vector of nodal displacement increments, $\{F\}$ the external load vector, and $\{I\}$ the vector of internal forces. When the response of the structure is nonlinear, the matrices $[K_T]$, $[K_G]$, and $[I]$ are functions of the displacement vector $\{X\}$. This is due to the nonlinear relationship between strains and displacements.

Because of its nonlinearity, the numerical solution to Eq. (15) can be obtained by using an iterative scheme. In order to investigate the nonlinear response of the dome structure in the next section, a finite element program, MAGNA,[8] was used. Of the different solution schemes available in the program, the full Newton-Raphson iteration technique was used.

Numerical Example

In order to investigate the nonlinear behavior of an optimized structure, the 30-member structure shown in Fig. 1 was optimized with different stability requirements. The nodal points of this structure lie on a spherical cap of radius 3060.00 in. (77.72 m). The elements of the structure are bars. The structure was subjected to a concentrated load of 1000 lb (453.59 kg) acting at nodes 1-7 in the negative Z direction. This is the design load. Nodes 8-19 are fixed. The coordinates of the node points in one quarter are given in Table 1. The coordinates of the other node points can be found by symmetry. The Young's modulus (E) of 10^7 psi (68.94 GPa) and material density (ρ) of 0.1 lb/in.^{3} (2768 kg/m^3) were used. The minimum size for all elements was set to 0.1 in.^{2} (0.65 cm^2) for this structure.

A near minimum weight design with constraints satisfied as inequality constraints was not difficult to obtain with a step size parameter r equal to 4. However, in order to satisfy the active constraints as equality constraints, the step size parameter was increased to 20. The number of iterations needed to obtain a minimum weight design was about 35 for all the problems. The initial weight with uniform areas for all members was 193.71 lb (87.86 kg). This design was designated as case I. Three constraint requirements were imposed on the structure and they are discussed below.

In case II the structure was optimized with $\alpha_j = 1.0$ in Eq. (2). This constraint condition requires the buckling loads associated with all the critical buckling modes to be equal at

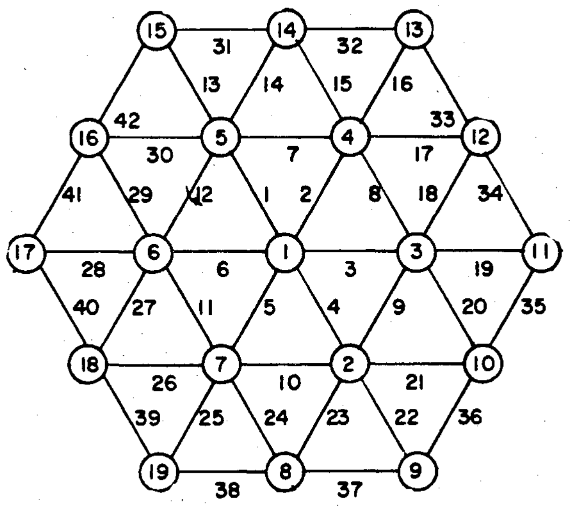

Fig. 1 Plan view of the three-dimensional dome structure. $E = 10^7$ psi (68.94 GPa); $P = 0.1$ lb/in.^{3} (2768 kg/m^3); minimum size = 0.1 in.^{2} (0.65 cm^2).

Table 1 Coordinates of the node points, in. (m)

Node	X	Y	Z
1	0.0	0.0	85.912(2.18)
3	360.0(9.14)	0.0	64.662(1.64)
4	180.0(4.57)	311.769(7.92)	64.662(1.64)
11	720.0(18.29)	0.0	0.0
12	540.0(13.72)	311.769(7.92)	21.709(0.55)
13	360.0(9.14)	623.538(15.83)	0.0
14	0.0	623.538(15.83)	21.709(0.55)

the optimum. For the dome structure it was found that there are two buckling modes which are critical at the optimum. In these buckling modes the displacements in the X and Y directions were very small compared to those in the Z direction. Therefore we will discuss only the displacements in the Z direction. In the first mode, node 1 practically did not move. Nodes 2, 4, and 6 moved in the positive Z direction, and nodes 3, 5, and 7 moved in the negative Z direction through the same amount forming a sinewave pattern (see Fig. 2). In the second mode, node 1 moved in the negative Z direction and the relative displacement of nodes 2-7 in the positive Z direction was 0.2917 that of node 1 (see Fig. 3). The eigenvalues associated with the two modes were 1.00000 and 1.00004. It was not possible to obtain a design with eigenvalues closer than these values. The weight of the optimum design was 168.70 lb (76.52 kg), and the maximum stress in an element for this design was 25,900 lb/in.^{2} (178.58 MPa). The areas of the members for the minimum weight design are given in Table 2.

In case III the dome structure was designed with $\alpha_1 = 1.0$ and $\alpha_2 = 1.1$, where α_1 is associated with the first mode and α_2 is associated with the second mode of case II. In the first mode (Fig. 2), node 1 was stationary, and nodes 2-7 had a sinewave pattern. In the second mode (Fig. 3), node 1 moved in the negative Z direction and the relative displacement of nodes 2-7 in the positive direction was 0.2842 that of node 1. The eigenvalues associated with the two modes for the optimum design were 1.00000 and 1.10006. The weight of the optimum design was 173.17 lb (78.54 kg). The areas of the members for this design are given in Table 2. The maximum stress in an element for this design was 26,375 lb/in.^{2} (181.86 MPa).

The structure in case IV was designed with $\alpha_1 = 1.0$ and $\alpha_2 = 1.1$, with the condition that α_1 be associated with the second mode (Fig. 3) and α_2 with the first mode (Fig. 2). The second mode (Fig. 3) was the critical one. Node 1 moved in the negative Z direction and the relative displacement of nodes 2-7 in the positive Z direction was 0.2977 that of node 1. In the noncritical mode associated with α_2, nodes 2-7 had a sinewave pattern. The weight of the optimum design was 181.39 lb (82.27 kg), and the maximum stress in an element was 23,000 lb/in.^{2} (158.58 MPa). The eigenvalues associated with the two modes were 1.00000 and 1.10000. The areas of the optimum design are given in Table 2. For this constraint condition it was not possible to design the dome structure with $\alpha_2 \geq 1.2$.

Comparing the three optimum designs, it is seen that $\alpha_1 = \alpha_2 = 1$ gave a least weight design. For the same value of α_2 equal to 1.1, case IV, with the second mode being the critical one, gave a higher weight design than case III. Case I, with uniform areas, in reality is not far from the optimum weight design. This can be seen by comparing the weights of all the designs. For uniform areas the first and second modes were the same as case III, and the eigenvalues associated with these modes were 1.0000 and 1.03575, respectively.

The nonlinear response of the four designs was determined by analyzing the structure using the computer program MAGNA.[8] The nonlinear critical load (P^*_{cr}) was determined within the accuracy of 0.5 lb (.2267 kg). The first load level was 100 lb (45.35 kg), and this was incremented with 50, 10, and 0.5 lb. (22.678, 4.535, .2267 kg) The number of increments depend on the magnitude of P^*_{cr}. For example, where P^*_{cr} was 392.5 lb (178.02 kg), the load increments were 50 lb (22.678 kg) for load levels from 100 to 350 lb (45.35 to 158.73 kg), 10 lb (4.535 kg) for load levels from 360 to 390 lb (163.29 to 176.90 kg), and then the increments were 0.5 lb (.2267 kg). For all the problems two or three computer runs had to be made in order to fix the range of the loads and specify the proper increments. MAGNA does not have an automatic capability to select the proper load increments. The nonlinear critical load was the last load level for which the nonlinear incremental equilibrium equations were solved correctly. P^*_{cr} is the critical load level at which the stiffness matrix ceases to be positive definite.

Each design was analyzed with five different initial geometric imperfections. They are given in Table 3 and are designated by I-1 through I-5. I-1 was the perfect structure. The geometric imperfections for I-2 and I-3 were similar to the first (Fig. 2) and second (Fig. 3) eigenvalue buckling modes for the case I and III designs. The I-4 imperfection was similar to the nonlinear failure mode of the perfect structure for the case II design. This mode of failure was similar for all the designs which were analyzed. I-5 has only a central node displaced in the negative Z direction. The displaced position of nodes 2-7 for all the imperfection patterns was in the negative Z direction and by the same amount except for I-2, where the displaced nodes form a sinewave pattern.

The load displacement curves for case II with no imperfections (I-1) are shown in Fig. 4. The displaced position of the nodes at P^*_{cr} is shown in Fig. 5. The imperfection pattern

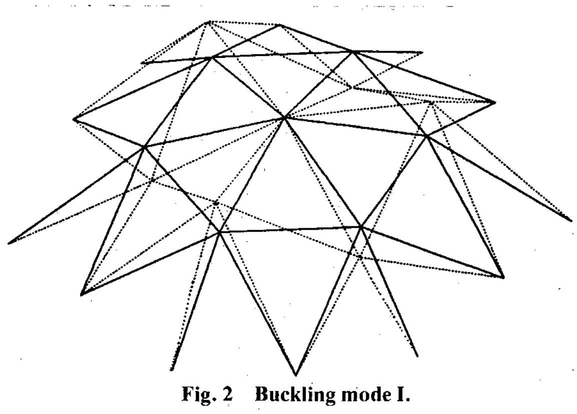

Fig. 2 Buckling mode I.

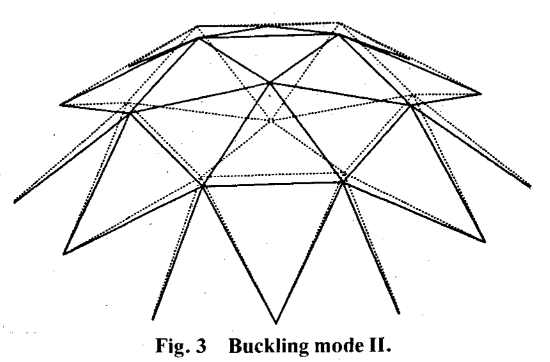

Fig. 3 Buckling mode II.

Table 2 Areas of the members for the minimum weight design, in.^{2} (10^{-3} m^2)

Elements	Case I	Case II	Case III	Case IV
1, 2, 3, 4, 5, 6	0.17823 (0.11496)	0.26609 (0.17167)	0.27141 (0.17510)	0.28531 (0.18407)
7, 8, 9, 10, 11, 12	0.17823 (0.11496)	0.13706 (0.08842)	0.15604 (0.10067)	0.13307 (0.08585)
13, 16, 19, 22, 25, 28	0.17823 (0.11496)	0.11551 (0.07452)	0.11640 (0.07509)	0.12763 (0.08234)
14, 15, 17, 18, 20, 21, 23, 24, 26, 27, 29, 30	0.17823 (0.11496)	0.12910 (0.08329)	0.12687 (0.08185)	0.14459 (0.09328)
Weight, lb (kg)	193.71 (87.86)	168.70 (76.52)	173.17 (78.54)	181.39 (82.28)

Table 3 Imperfections[a]

Imperfection shape	Node 1	Nodes 2, 4, 5	Nodes 3, 5, 7
I-1 (No imperfections)	0.0	0.0	0.0
I-2	0.0	+0.25 (+0.63)	−0.25 (−0.63)
I-3	−0.5 (−1.27)	+0.15 (+0.38)	+0.15 (+0.38)
I-4	−0.1 (−0.25)	−0.5 (−1.27)	−0.5 (−1.27)
I-5	−0.5 (−1.27)	0.0	0.0

[a]Changes in the Z coordinates in inches (10^{-4} m).

for I-4 is similar to this one. The nonlinear critical loads (P^*_{cr}) and the vertical displacements of nodes 1-7 for the five imperfection patterns of the four designs (cases I-IV) are given in Tables 4-7. It may be pointed out here again that the design load, i.e., the linear eigenvalue buckling load, for all four cases was 1000 lb (453.59 kg).

Comparing the data given in Tables 4-7, the following observations were made.

1) The critical displacement pattern for all cases was the same. The displacements of nodes 2-7 were substantially larger than node 1. The displacements of nodes 2-7 were nearly the same except for case I (uniform member areas, Table 4) where a small sinewave pattern was noticed. The displacements of nodes 2-7 for cases II-IV were slightly larger than for case I.

2) The critical displacement of the middle node for all the imperfection patterns was smaller than the surrounding nodes. The displacement of the middle node was in general maximum for case I (uniform member areas design) and a minimum for case IV.

3) The relative influence of the imperfections on the critical buckling load was the same for all cases. If we write the imperfection pattern in order, so that the first pattern causes a maximum reduction in the critical load and the last one has the minimum effect, then the order would be I-2, I-4, I-5, and I-3. The imperfection patterns I-2 and I-3 are associated with the linear eigenvalue buckling modes; I-4 is similar to the nonlinear critical displacement pattern; and I-5 is the displaced position of node 1 only. It is interesting to note that the sinewave pattern (I-2) had a more degrading effect than the pattern (I-4) which was similar to the nonlinear critical displacement mode.

4) For case IV the critical eigenvalue buckling mode was the second mode where the maximum displacement was at node 1. The nonlinear displacement at node 1 for this case was smaller than for all other cases except for imperfection pattern I-2. For I-2 for case II, the displacement of node 1 was smaller than for case IV. This may be because the cross-sectional areas of members 1-6 for this case are larger than for the other cases (see Table 2).

5) The percentage drop from the design load (1000 lb) (453.59 kg) to P^*_{cr} was predominantly due to the nonlinear behavior rather than the effect of the imperfections. This can be seen by comparing P^*_{cr} values for the perfect structure (I-1) to the imperfect structure (I-2 through I-4) and the design load (1000 lb) (453.59 kg).

6) The nonlinear critical load with the I-3 and I-5 imperfection pattern was larger than the perfect structure (I-1).

The dome structure was idealized with bar elements. For this case the tangent stiffness matrix and the geometric

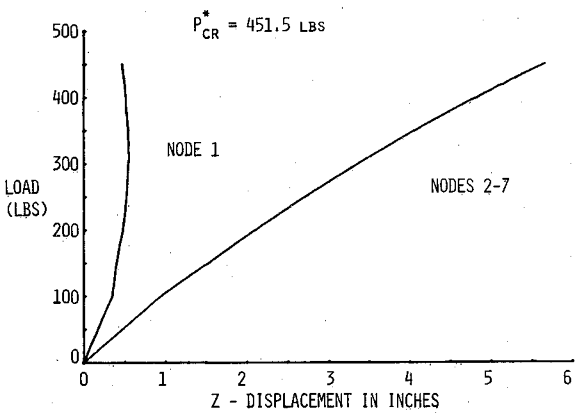

Fig. 4 Nonlinear load displacement curve for case II with no imperfections.

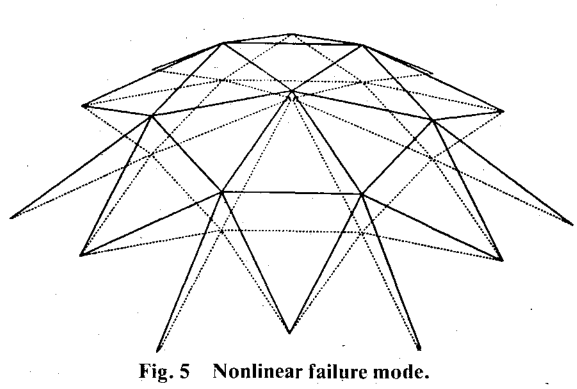

Fig. 5 Nonlinear failure mode.

Table 4 Nonlinear analysis (case I)

Imperfection shape	P^*_{cr}, lb (kg)	Displacement at nodes, in. (10^{-2} m) at P^*_{cr}: 1	2, 4, 6	3, 5, 7
I-1	578.00	2.8076	4.9381	4.9338
	(262.2)	(0.0713)	(0.1254)	(0.1253)
I-2	492.50	2.0756	2.7786	5.8306
	(223.4)	(0.0527)	(0.0705)	(0.1480)
I-3	605.50	3.1895	5.1056	5.1089
	(274.6)	(0.0810)	(0.1296)	(0.1298)
I-4	545.00	2.5303	4.7826	4.7526
	(247.2)	(0.0642)	(0.1214)	(0.1207)
I-5	593.50	3.0533	5.0763	5.0163
	(269.2)	(0.0775)	(0.1289)	(0.1274)

Table 5 Nonlinear analysis (case II)

Imperfection shape	P^*_{cr}, lb (kg)	Displacement at nodes, in. (10^{-2} m) at P^*_{cr}: 1	2, 4, 6	3, 5, 7
I-1	451.5	0.4665	5.6518	5.6518
	(204.7)	(0.0118)	(0.1435)	(0.1435)
I-2	392.0	0.2036	3.4427	6.4979
	(177.81)	(0.0052)	(0.0874)	(0.1650)
I-3	465.0	0.4356	5.7811	5.7811
	(210.9)	(0.0111)	(0.1468)	(0.1468)
I-4	427.0	0.4862	5.4392	5.4392
	(193.68)	(0.0123)	(0.1381)	(0.1381)
I-5	458.0	0.4432	5.7290	5.7290
	(207.75)	(0.0112)	(0.1455)	(0.1455)

Table 6 Nonlinear analysis (case III)

Imperfection shape	P^*_{cr}, lb (kg)	Displacement at nodes, in. (10^{-2} m) at P^*_{cr}: 1	2, 4, 6	3, 5, 7
I-1	474.5	1.1188	5.5951	5.5951
	(215.2)	(0.0284)	(0.1421)	(0.1421)
I-2	411.0	0.7183	3.3081	6.6110
	(186.4)	(0.0182)	(0.0840)	(0.1679)
I-3	489.0	1.1337	5.7332	5.7332
	(221.8)	(0.0287)	(0.1456)	(0.1456)
I-4	451.5	1.0962	5.4369	5.4368
	(204.7)	(0.0278)	(0.1380)	(0.1380)
I-5	481.5	1.1256	5.6799	5.6799
	(218.4)	(0.0285)	(0.1442)	(0.1442)

Table 7 Nonlinear analysis (case IV)

Imperfection shape	P^*_{cr}, lb (kg)	Displacement at nodes, in. (10^{-2} m) at P^*_{cr}: 1	2, 4, 6	3, 5, 7
I-1	472.5	0.1697	5.6962	5.6962
	(214.3)	(0.0043)	(0.1446)	(0.1446)
I-2	412.0	0.3786	3.4516	6.6709
	(186.8)	(0.0096)	(0.0876)	(0.1694)
I-3	487.0	0.2490	5.8363	5.8361
	(220.9)	(0.0063)	(0.1482)	(0.1482)
I-4	451.5	0.1169	5.5572	5.5572
	(204.7)	(0.0029)	(0.1412)	(0.1412)
I-5	479.5	0.2196	5.7800	5.7800
	(217.4)	(0.0056)	(0.1468)	(0.1468)

stiffness matrix are linear functions of the design variables A_i. Thus, if we increase the areas by a certain factor, then the nonlinear critical load will also increase by the same factor. Hence, if the areas of all the designs are multiplied by the ratio 1000 lb (453.59 kg) to P^*_{cr}, then the nonlinear critical loads of the scaled designs will be equal to the design load of 1000 lb (453.59 kg). In Table 8, the weight of the structure for the scaled design is given for the four cases and five imperfection patterns. The weight of the structure where the linear eigenvalue buckling load is equal to the design load is also given for comparison.

Comparing the weights for the different cases, the following observations can be made.

1) The uniform area design has the minimum weight for any given imperfection pattern. It should be emphasized here that this will not necessarily be true for some other type of

Table 8 Weights of the scaled optimum designs for $P^*_{cr} = 1000$ lb (453.29 kg), lb (kg)

Imperfection pattern	Case I	Case II	Case III	Case IV
I-1	335.14 (152.02)	373.64 (169.48)	364.95 (165.53)	383.89 (174.13)
I-2	393.32 (178.40)	430.35 (195.20)	421.34 (191.11)	440.26 (199.69)
I-3	319.91 (145.10)	362.79 (164.55)	354.13 (160.63)	372.46 (168.94)
I-4	355.43 (161.22)	395.08 (179.20)	383.54 (173.97)	401.75 (182.23)
I-5	326.38 (148.04)	368.00 (166.92)	359.64 (163.13)	378.28 (171.58)
Linear eigenvalue buckling load	193.71 (87.86)	168.70 (76.52)	173.17 (78.54)	181.39 (82.27)

load. The uniform area design is a good design for this structure because of the symmetry and uniformity of the load applied.

2) The weight of the designs for case III ($\alpha_1 = 1.0$, $\alpha_2 = 1.1$) is lower than that for case II ($\alpha_1 = \alpha_2 = 1.0$). The case IV where the buckling mode was switched did not improve the load carrying capacity of the structure.

3) The imperfection patterns I-3 and I-5 were beneficial in increasing the load carrying capacity of the structure and reducing the weight.

The observations made in this paper are based on the nonlinear analysis of an optimized structure which is designed to satisfy constraints on system stability defined by an eigenvalue problem. The correct procedure for this type of structure would be to optimize it based on its nonlinear response which requires the development of a new optimization algorithm. This need is going to increase particularly when we enter into the era of space structures. An algorithm based on optimality criterion approach using nonlinear analysis is discussed in Ref. 9.

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